RESEARCH DESCRIPTION

Ionel POPESCU

I am a probabilist but I consider myself an analyst at large. My interest covers stochastic analysis on manifolds, random matrices and free probability.

I did my undergraduate in Romania with a diploma on non-commutative L^p spaces under the coordination of Ioan Cuculescu. Ironically, this is more operator theory, though I was dealing quite a lot with probability theory. As a master student I worked closer to Şerban Strătilă in the free probability world and I did a master thesis on the free Brownian motion and free stochastic calculus, partly under the guidance of Roland Speicher who was at that time in Heidelberg. This is just the parallel of the classical stochastic calculus though worked out in a non-commutative context.

When I got to MIT, where I did my Ph.D, I decided to work with Dan Stroock and the subject he suggested to me was a probabilistic approach of Morse Inequalities (and possibly Morse theory) using probabilistic techniques. There is a new point of view on Morse theory which was initiated by Witten and is based on the idea that given a Morse or Bott-Morse function, one can construct a deformation of the De-Rham complex with the differential $d^\alpha = e^{\alpha h} d e^{-\alpha h}$. Using this one can construct the operator $\Box^\alpha = (d^\alpha + \delta^\alpha)^2$ which is known as the Witten Laplacian. Here δ^α is the adjoint of the d^α provided that one chooses a Riemannian metric on the manifold in discussion. The idea is that for $\alpha = 0$, d^α is nothing but the usual De-Rham differential while for any α the cohomology of the manifold with the operator d^α on forms does not change with α. Now, letting α tend to ∞ one ends up with a complex which is given in terms of the critical points of the function h. This is what I had to do using probability. It seems amenable to probability due to the fact that the operator involved is sort of Laplacian and this is known to be intimately related to the Brownian motion.

Ricci flow is well known for its contribution to the resolution of the Poincaré and the geometrization conjectures and is a very exciting area of research. This flow, as well as other flows (for instance mean curvature) are interpreted as a heat equation type evolution for the metric on the manifold and as it is well known, this ought to have a probabilistic interpretation. One of the most interesting aspects of the Ricci flow is that it tends to round the metric. For instance, on surfaces the normalized Ricci flow exists for all time and converges to a metric of constant curvature. Together with Rob Neel I obtained an entirely probabilistic proof that on surfaces of non-positive Euler characteristics the metric under the Ricci flow converges exponentially fast with all its derivatives to a metric of constant curvature with some estimates better than those in the Ricci flow literature. This is certainly the first approach which is entirely different from the geometric approach of the Ricci flow literature and I have no doubt that it will be very fruitful in both the geometric and probabilistic community.

The ideas involved there exploits on the fact that the Ricci flow on surfaces is essentially driven by a conformal factor and this satisfies a non-linear type equation. We represent the solution via a stochastic target problem and then analyze it using stochastic analysis more specifically techniques from couplings. I must mention that these tools are perhaps among the very few purely probabilistic tools which can not be replicated by analytic tools in a natural way. Typically the coupling works for getting gradient estimates of various evolutions equations but we use it two folds. One for estimating the gradient but before that for estimating the oscillation of the conformal factor. This is perhaps sort of novelty. The other novelty is that we use a coupling of three particles which turns out to give a Hessian estimate, something which is particularly hard to obtain directly for non-linear equations.

One of the main tools in probability for geometric and analytic problems is couplings of Brownian motions. On manifolds, one of the most useful coupling with a multitude of applications to geometric as well analytic problems is the Cranston-Kendall mirror coupling. This coupling is in some sense one which forces the Brownian motions to meet in finite time. On the other hand, Burdzy and others put forward a different notion of couplings, which is shy couplings, a version of couplings which are staying apart for all times. For instance, on bounded convex domains, there are no shy couplings of reflected Brownian motions. With Mihai Pascu I created on Riemannian manifolds with $Ric \geq 0$ (plus other technical conditions) a coupling of Brownian motions which stays fixed distance for all times. This curious and intriguing coupling is a strong version of shy coupling in the first place, but in the second place seems to be a promising path toward the Hot Spots conjecture which states that the second Newman eigenfunction on convex domains attains its maximum on the boundary.

After the Ph.D., I started to look back at free probability, and at the same time to a new and interesting subject, namely mass transportation. This latter subject is at the intersection of economics (where it actually started) pde, probability, functional inequalities and geometry as is well described by Villani in his famous book. On the other hand, free probability was initially related to operator algebras which was the original interest of Voiculescu, but is now used in a variety of areas, as for instance random matrices, engineering, numerical analysis and even finance.

A particular area of intensive research in the classical probability is functional inequalities, as for instance Log-Sobolev, isoperimetric inequalities, which are motivated by geometry, pde, Markov processes, etc. In this framework, free functional inequalities arise naturally as limiting of their classical counterparts using large random matrix approximations combined with the corresponding classical functional as for instance in but these techniques work in a limited number of cases. I was the first one to use mass transportation tools to prove one-dimensional free transportation inequality in a paper by myself and later on with Michel Ledoux to give a very simple proof with sharp constants of the free Log-Sobolev and propose and prove a refinement of these inequalities. We further investigated the free Poincaré inequality in in connection with the other free functional inequalities and the intriguing part is that it has a very different spirit from its classical counterpart and in some sense is very closely related to the universality of fluctuations of random matrices. For part of this work I was awarded by Romanian Academy for contributions to "Free and Non-Commutative Probability". This is an area in which I have several other problems to work on which will deliver exciting results.

In a recent paper, Maida and Maurel-Segala published in PTRF (one of the two best journals in probability) proposed a transportation inequality for logarithmic energy with potentials which are no longer convex. Their approach gave a constant in the inequality which depended in a very implicit way on the potential. After they posted the paper I realized that I can do much better and wrote a paper with a version of the transportation on compact intervals which is independent of the potential. They acknowledge my approach in their paper by saying "This version of the Theorem which can be seen as a local one (it only gives information for μ and ν living in a given compact set K) has since been recovered with completely different methods by Popescu in [31] with an optimal constant depending only on the size of K."

I have in work several interesting extensions of the free analogues of classical inequalities, as for instance the transportation and Log-Sobolev on the circle. These seems to be challenging as the Wasserstein metric which is obtained through various heuristic unsung random matrix models does not seem to be the correct one. In another paper which close to being submitted I have some extensions to the free Poincare inequality. Some are of the Brascamp-Lieb flavor and the other is related to an inequality introduced by Houdre in the classical case. Also very intriguing is the notion of the mass concentration phenomenon which is very different from the classical one as for instance there is a certain amount of mass outside of large balls. Here the measure involved is compactly supported and it seems that the phenomena takes place at the level of logarithmic capacity of sets. This is an ongoing project with Michel Ledoux. The more challenging is to give more in the direction of several non-commuting variables, as for instance transportation or Log-Sobolev using either of the entropies introduced by Voiculescu. Even though the random matrix theory seems to suggest that something like this is possible, so far these are not established at all.

The random matrix theory is one of the fast expanding area of research which has connections with, physics, combinatorics, integrable systems, statistics, operator algebras, number theory, finance, engineering and so on. Dumitriu and Edelman introduced a tridiagonal model for the so called β-ensemble which in the case of $\beta = 1, 2, 4$ are naturally obtained as tridiagonalization of the GOE, GUE, GSE ensembles. They used this particular structure to study the limiting distribution of the eigenvalues which is still a semicircular for any $\beta > 0$. The random matrix models with Gaussian entries have a natural generalization into Wigner ensembles in which the entries are independent but not Gaussian. In the world of tridiagonal matrices I introduced and studied the limiting distribution and the fluctuations of a general tridiagonal model. The techniques are related to what is called the moment problem and the idea is relatively simple, though the counting could become delicate in other models. This was expanded further by other people in some other cases but there are open questions in this area which are suitable not only to experts in the field but also to students who can easily pick up the problem and then work and expand on it.

In a different direction, but also in random matrix models related to combinatorics and enumeration of planar diagrams, together with Stavros Garoufalidis I solved a conjecture of 't Hooft's about the radius of convergence of the generating function of planar diagrams. This topic of research is a very active one particularly because of its relation to topological string theory as it is pointed in the physics literature. Our approach is relatively elementary and is just a combination of magics of Chebyshev polynomials, combinatorics and properties of generating functions of sequences which satisfy algebraic functions.

The areas I work on seem different but are interconnected. I prefer a set of diverse problems because interdisciplinary is more likely to lead to breakthroughs and deeper results.